Supporting Information for **RSI20-AR-02824**

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1. Derivation of Equation (6), (7) and (8) in the main text from the work of Reference 21

The reflection matrix for a forward-propagating beam incident on a sample is given by[21]

$$M_R^{(f)} = \begin{bmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{bmatrix} = \begin{bmatrix} r_p + r_{pp,T} & r_{ps,L} + r_{ps,P} \\ r_{sp,L} + r_{sp,P} & r_s \end{bmatrix} = \begin{bmatrix} r_p + \alpha_x m_x & \alpha_y m_y + \alpha_z m_z \\ -\alpha_y m_y + \alpha_z m_z & r_s \end{bmatrix} \qquad \text{(S-1)}$$

At an incidence angle $\theta_{inc} = 20°$, reflectivities for *s*-polarization and *p*-polarization have nearly the same amplitude and are 180° out of phase, i.e., $r_s = -\, r_p$.

Longitudinal Kerr angle $\boldsymbol{\theta_{K,L}}$ By using a half-wave plate for the first wave plate with its fast axis (FA) at 22.5° from the *p*-polarization (so that $a = b = 1/\sqrt{2}$ and $\varphi = 0$) and using a quarter-wave plate for the second wave plate with its FA parallel to the *p*-polarization after the sample, Equation (18) in Reference 21 yields the following

$$\alpha_K = Im\left\{\frac{4\left(r_{ps,L}(r_p - r_s) + r_{ps,P}(r_p + r_s)\right)e^{i\varphi}}{(r_p^2 + r_s^2 e^{i2\varphi})}\right\} \cong Im\left\{\frac{4r_{ps,L}}{r_p}\right\} = Im\left\{\frac{4\alpha_y m_y}{r_p}\right\} \equiv 2\theta_{K,L} \qquad \text{(S-2)}$$

$$\theta_{K,L} \cong Im\left\{\frac{2r_{ps,L}}{r_p}\right\} = Im\left\{\frac{2\alpha_y m_y}{r_p}\right\} \qquad \text{(S-3)}$$

$r_{ps,L}$ is given in Table II by Hunt in Reference 9. At $\theta_{inc} = 20°$, we find

$$\frac{r_{ps,L}}{r_p} \cong \frac{iQ sin\theta_{inc}}{\varepsilon_s - 1} \qquad \text{(S-4)}$$

(S-3) and (S-4) yield Equation (6) in the main text: $\theta_{K,L} \cong Re\left\{\frac{2sin\theta_{inc}Q}{\varepsilon_s - 1}\right\} m_y$.

Transverse Kerr angle $\boldsymbol{\theta_{K,T}}$ By removing the first wave plate (so that $a = 1$, $b = 0$ and $\varphi = 0$) and using a quarter-wave plate for the second wave plate with its FA set to be + 45° rotated from the *p*-polarization after reflection from the sample, Equation (21) in Reference 21 yields

$$\alpha_K = Im\left\{\frac{2r_{pp,T}}{r_p}\right\} = Im\left\{\frac{2\alpha_x m_x}{r_p}\right\} \equiv 2\theta_{K,T} \qquad \text{(S-5)}$$

$$\theta_{K,T} = Im\left\{\frac{r_{pp,T}}{r_p}\right\} = Im\left\{\frac{\alpha_x m_x}{r_p}\right\} \qquad \text{(S-6)}$$

$r_{pp,T}$ is given in Table I by Hunt in Reference 9. At $\theta_{inc} = 20°$, we find

$$\frac{r_{pp,T}}{r_p} \cong \frac{i2Qsin\theta_{inc}}{\varepsilon_s - 1} \tag{S-7}$$

(S-6) and (S-7) yield Equation (7) in the main text: $\theta_{K,T} \cong Re\left\{\frac{2sin\theta_{inc}Q}{\varepsilon_s - 1}\right\} m_x$.

Polar Kerr angle $\boldsymbol{\theta_{K,P}}$ By choosing a quarter-wave plate for the first wave plate with its FA set to + 45° from the *p*-polarization (so that $a = b = 1/\sqrt{2}$ and $\varphi = 90°$) and removing the second wave plate entirely, Equation (15) in Reference 21 yields

$$\alpha_K = Im\left\{\frac{4\left(r_{ps,L}(r_p + r_s) + r_{ps,P}(r_p - r_s)\right)e^{i\varphi}}{(r_p^2 - r_s^2 e^{i2\varphi})}\right\} \cong Im\left\{\frac{4ir_{ps,P}}{r_p}\right\} = Im\left\{\frac{4i\alpha_z m_z}{r_p}\right\} \equiv 2\theta_{K,P} \tag{S-8}$$

$$\theta_{K,P} \cong Im\left\{\frac{2ir_{ps,P}}{r_p}\right\} = Im\left\{\frac{2i\alpha_z m_z}{r_p}\right\} \tag{S-9}$$

$r_{ps,P}$ is given in Table II by Hunt in Reference 9. At $\theta_{inc} = 20°$, we find

$$\frac{r_{ps,P}}{r_p} \cong -\frac{iQ\sqrt{\varepsilon_s}}{\varepsilon_s - 1} \tag{S-10}$$

(S-9) and (S-10) yield Equation (8) in the main text: $\theta_{K,P} \cong Im\left\{\frac{2\sqrt{\varepsilon_s}Q}{\varepsilon_s - 1}\right\} m_z$.

2. Scan head for the oblique-incidence Sagnac interferometric scanning microscope: